Inhomogeneous distribution particles in self-gravitational system.

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The microcanonical partition function for self-gravitational system in three dimensional case has been found. Used approach from the field theory of statistical description of the system was tailored to gravitational interacting particles with regard for an arbitrary spatially inhomogeneous particle distribution. The entropy of self-gravitational system has been found from extreme condition for the effective functional. For inhomogeneous distribution particle (formation few cluster of finite size) the entropy are bigger as the entropy homogeneous distribution of particles. The increasing of entropy of self-gravitational system after formation cluster motive tendency to disperse.

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The study of self-gravitational system has as fundamental as practical physical interest. The self-gravitational system are interesting for testing ideas about the statistical mechanic description of systems governed by long range interaction. A self-gravitational system has also more general problem, studied for a long time [2]. The standard methods of statistical mechanic cannot be carrier to study gravitational system. Due to this fundamental difference, the notion of equilibrium is not always well defined and those system exhibit a nontrivial behaviour with gravitational collapse. For system with gravitational interaction the thermodynamically ensemble are inequivalent, negative specific heat [3] in microcanonical ensemble which not exist in canonical description [4].In the microcanonical ensemble self collapse correspond a "gravithermal catastrophe" and in canonical ensemble to an " isothermal collapse" [4]. A self gravitational system can increase entropy without bound by developing a dense and hot core surrounded by dilute halo. Since equilibrium states are only local entropy maximum. However, if introduce a repulsive potential at short distance, complete core collapse is prevented and can to proved that a global entropy maximum now exist for all accessible values of energy. The effective repulsion can be introduce in many different way but the physical results are rather insensitive to the precise form of regularization. Alternative can consider a classical hard-sphere gas by introducing excluded volume around each particle [7]. For the gas with pure gravitational interaction between its particles the virial coefficients for potential of particles interaction $1/r^n$ if $n < 3$ and thus the partition function diverges, the energy of gravitating particle will not be an extensive parameter. The entropy such system tends to infinity too when volume $V \to \infty$. In it was shown that a self-gravitating gas collapses. A nature of the collapse and its conditions are explained by using simple and clear consideration [8]. The phase transition in such systems creates the problem of description in the mean-field thermodynamics approach [4]. Two type approaches (statistical and thermodynamic) have been develop to determination the equilibrium states of self-gravitational system [4], [5]. About all this problem and possible solution very good described in review [6]. The collapse in such system are begin as spatially homogeneous distribution of particles in the all system at once. Formation of the spatially inhomogeneous distribution of interaction particles is a typical problem in condensed matter physics and requires non-conventional methods of statistical description of the system was tailored to gravitational interacting particles with regard for an arbitrary spatially inhomogeneous particle distribution. This method must employs the procedure to find dominant contributions to the partition function and to avoid entropy divergences for infinite system volume. Only a few model systems with interaction are known for which the partition function can be exactly evaluated, at least within thermodynamic limits [9] but not for inhomogeneous distribution of particle. As for existence of equilibrium states, a few result have been obtained in the framework of "exact" equilibrium statistic mechanics with one considered a finite number of particles [10]. Is no known exact solution in even in three dimensional self- gravitational system. Formation of the spatially inhomogeneous distribution of interaction particles requires a nonconventional method, such as use in [11], [12], [13], which are based on Hubbard-Stratonovich representation of statistical sum [14]. This method is now extended and applied to a gravitational interacting system to find solution for particles distribution without using spatial box restrictions. It is important that this solution has no divergences in thermodynamic limits. For this goal can use saddle point approximation which take into account the conservation number of particles in limiting space, which provided to nonlinear equation. Described condition give the possibility determine microcanonical partition function for self- gravitational system. This partition function in the case homogeneous distribution of particle, and in case inhomogeneous distribution with formation few equal cluster with same size has not any peculiarity and fully determined thermodynamic parameter of system. Has been shown, that the entropy for inhomogeneous distribution of particle in self-gravitational system are bigger as entropy homogeneous distribution of particle. This thermodynamic force, which appear by formation spatial inhomogeneous distribution of particle can be motivation of motion of cluster.

In particular can consider the microcanonical evolution of self-gravitational system. In the microcanonical ensemble, the object of fundamental interest is the density of states, which can written in the standard form [15]

$$\Omega_{E,N} = \int d\mathbf{q}d\mathbf{p}\delta(H(\mathbf{p},\mathbf{q}) - E)\delta(N(\mathbf{p},\mathbf{q}) - N) \tag{1}$$

where $\Omega_{E,N}$ denote microcanonical partition function for fixed energy and number of particles in system. Introduce the Lagrange multiplier $\beta = \frac{1}{kT}$ inverse temperature and $\eta = \beta\mu$ where μ-is chemical potential and make Laplase transformation can present the microcanonical distribution function in the form :

$$\Omega_{E,N} = \oint d\beta \oint d\eta \exp\left\{\beta E - \eta N\right\} \int d\mathbf{q}d\mathbf{p} \exp\left\{-\beta H(\mathbf{p},\mathbf{q}) + \eta N(\mathbf{p},\mathbf{q})\right\} \tag{2}$$

The entropy and temperature for microcanonical ensemble are defined by $S = \ln \Omega_{E,N}$ and inverse temperature can determined from relation $\beta = \frac{dS}{dE}$. The pressure of system defined through formula $P = \frac{1}{\beta}\frac{\partial S}{\partial V}_E$ and present the equation of state for system.

A system of interacting particles can be treated in the classical manner as Ising model with Hamiltonian [16] present in term occupation number in the form:

$$H(n) = \sum_s \varepsilon_s n_s - \frac{1}{2}\sum_{s,s'} W_{ss'} n_s n_{s'} \tag{3}$$

where ε_s is the additive part of the energy in the state s which is equal in most cases to the kinetic energy [16], $W_{ss'}$ are interaction energy for the particles in the states s and s'. In this model the macroscopic states of the system are described by a set of occupation numbers n_s. Index s labels an individual particle state; and can correspond as well as a fixed site on the Ising lattice [17], which explicit form is irrelevant in the continuum approximation. The number of particle is fixed which can determine from relation $N(n) = \sum_s n_s$. It is clear that calculating of the partition function as a rather complicated problem even in the case of the Ising model. The microcanonical partition function of a system of interacting particles can given in the form [12]:

$$\Omega_{E,N} = \oint d\beta \oint d\eta \exp\left\{\beta E - \eta N\right\} \sum_{\{n\}} \exp\left(-\beta H(n)\right) \tag{4}$$

or explicit definition form

$$\Omega_{E,N} = \oint d\beta \oint d\eta \exp\left\{\beta E - \eta N\right\} \sum_{\{n\}} \exp\left\{-\beta\left[\sum_s \varepsilon_s n_s - \frac{1}{2}\sum_{s,s'} W_{ss'} n_s n_{s'}\right]\right\} \tag{5}$$

where $\sum\limits_{\{n\}}$ implies summation over all probable distributions $\{n_s\}$. In order to perform a formal summation, additional field variables can be introduced making use of the theory of Gaussian integrals [14], [16]:

$$\exp\left\{\frac{1}{2\theta}\nu^2 \sum_{s,s'} \omega_{ss'} n_s n_{s'}\right\} = \int\limits_{-\infty}^{\infty} D\varphi \exp\left\{\nu \sum_s n_s \varphi_s - \frac{1}{2\beta}\sum_{s,s'} \omega^{-1}_{ss'} \varphi_s \varphi_{s'}\right\} \tag{6}$$

where $D\varphi = \frac{\prod_s d\varphi_s}{\sqrt{\det 2\pi\beta\omega_{ss'}}}$ and $\omega^{-1}_{ss'}$ is the inverse of the interaction matrix. The latter satisfies the condition $\omega^{-1}_{ss''}\omega_{s''s'} = \delta_{ss'}$ with $\nu^2 = \pm 1$ depending on the character of interaction energy. If introduce instead of chemical potential other important variable - chemical activity $\xi \equiv e^{\beta\mu} = e^{\eta}$, and used the obvious relation $d\eta = \xi^{-1}d\xi$ the microcanonical partition function of a system of interacting particles may be rewritten as :

$$\Omega_{E,N} = \oint d\beta \oint d\xi \int\limits_{-\infty}^{\infty} D\varphi \exp\left\{\beta E - N\ln\xi - \frac{1}{2\beta}\sum_{s,s'}\left(W^{-1}_{ss'}\varphi_s\varphi_{s'}\right)\right\} \prod_s \left\{\xi \exp(-\beta\varepsilon_s + \varphi_s)\right\}^{n_s} \tag{7}$$

This microcanonical partition function can be used for calculating all thermodynamically properties of the system with fixed total number of particles and energy of system. Only in this presentation can make summation over occupation number. After summation over the occupation numbers n_s [12], microcanonical partition function finally reduces to:

$$\Omega_{E,N} = \oint d\beta \oint d\xi \int\limits_{-\infty}^{\infty} D\varphi \exp(\beta S_{eff}(\varphi, \xi)) \tag{8}$$

where can introduce the effective entropy

$$S_{eff}(\varphi, \xi) = \frac{1}{2\beta} \sum_{s,s'} \left(W_{ss'}^{-1} \varphi_s \varphi_{s'} \right) - \delta \sum_s \ln\left(1 + \xi e^{\varphi - \beta\varepsilon_s}\right) + (N+1)\ln\xi - \beta E \tag{9}$$

where $\xi \equiv e^{\beta\mu}$ is absolute chemical activity of chemical potential μ. For this calculation was use the Fermi statistic,because the two classical particle can not occupied one spatial place. After this we can present partition function in the usual form $\Omega_{E,N} = \exp S$, where S is entropy of system. For this goal, for determination microcanonical partition function allows use the of efficient methods developed in the quantum field theory without imposing additional restrictions of integration over field variables or the perturbation theory. The functional $\beta S_{eff}(\varphi, \lambda)$ depends on distribution of the field variables φ and the absolute chemical activity ξ. The field variable φ contains the same information as original partition function with summation of over occupation numbers, i.e. all information about possible states of the systems. The saddle point method can now be further employed to find the asymptotic value of the partition function $\Omega_{E,N}$ for $N \to \infty$; the dominant contribution is given by the states which satisfy the extreme condition for the functional. The particles distribution is determined by the saddle point solutions of equations:

$$\frac{\delta S_{eff}}{\delta\xi} = \frac{\delta S_{eff}}{\delta\varphi} = 0 \tag{10}$$

whether this distribution of particles is spatially inhomogeneous or not. The solutions which correspond the finite entropy $S_{eff}(\varphi, \lambda)$ while the volume of the system tends to infinity, mean such solutions could be thermodynamically stable. The above set of equations in principle solves the many-particle problem in thermodynamic limit. The spatially inhomogeneous solution of this equations corespondent the distribution of interacting particles. Such inhomogeneous behavior is associated with the nature and intensity of interaction. In other words, accumulation of particles in a finite spatial region (formation of a cluster) reflects the spatial distribution of the fields and the activity. The inverse matrix $\omega_{ss'}^{-1}$ of the interaction, $\omega_{ss'} = \omega\left(|r_s - r_{s'}|\right)$, in continuum limit should be treated in the operator sense [16], i.e.

$$\omega_{rr'}^{-1} = \delta_{rr'} \widehat{L_{r'}} = -\frac{1}{4\pi G m^2} \triangle \tag{11}$$

where m is the mass of particle and $\triangle$- Laplace operator. With precision to surface term in continuum case the effective entropy takes form:

$$S_{eff}(\varphi, \xi) = \int dV \left\{ \frac{1}{8\pi G m^2 \beta} (\nabla\varphi)^2 + \delta \sum_p \ln\left(1 - \delta\xi e^{\varphi - \beta\varepsilon_p}\right) \right\} + N\ln\xi - \beta E \tag{12}$$

As shown before [11], in all cases classical Boltzmann statistic for high temperature $\xi \leq 1$ and can use expansion $\sum_p \ln\left(1 + \xi e^{\varphi - \beta\sum_p \varepsilon_p}\right) \approx \xi e^{\varphi - \beta\varepsilon_p} + ...$ Integration over the impulse and coordinates should be performed with regard for the cell volume $(2\pi\hbar)^3$ in the phase space of individual states [12]. After integration over the impulse the effective entropy can present in general form

$$S_{eff}(\varphi, \xi) = \int dV \left\{ \frac{1}{8\pi G m^2 \beta} (\nabla\varphi)^2 - \xi A e^{\varphi} \right\} + N\ln\xi - \beta E \tag{13}$$

where $A \equiv \left(\frac{2\pi m}{\beta\hbar^2}\right)^{\frac{3}{2}}$. The obtained microcanonical partition function can use to calculation all thermodynamically relation for self-gravitational system. At most general case it is impossible, but for individual case can obtain the all thermodynamic characteristic of system. Next will be examine separate case.

First from all we consider the case of system noninteracting particles, when $\varphi = 0$. The effective entropy in this case can write in the simple form

$$S_{eff}\left(\varphi,\xi\right) = -\int dV\xi\left(\frac{2\pi m}{\beta\hbar^2}\right)^{\frac{3}{2}} + N\ln\xi - \beta E \tag{14}$$

The extreme condition $\frac{\delta S_{eff}}{\delta\xi} = \frac{\delta S_{eff}}{\delta\beta} = 0$ reduce to two equations

$$V\xi\left(\frac{2\pi m}{\beta\hbar^2}\right)^{\frac{3}{2}} = N \tag{15}$$

and

$$V\xi\left(\frac{2\pi m}{\beta\hbar^2}\right)^{\frac{3}{2}} = \frac{2}{3}\beta E \tag{16}$$

from which at once determination the chemical activity

$$\xi = \frac{N}{V})\left(\frac{2\pi m}{\beta\hbar^2}\right)^{-\frac{3}{2}} \tag{17}$$

and well-know relation between fixed energy and number of particles in system and inverse temperature

$$\beta = \frac{3}{2}\frac{N}{E} \tag{18}$$

or $\frac{3}{2}NkT = E$. If determined coefficient substitute in to effective entropy can obtain the ordinary entropy

$$S_{E,N} = \ln\frac{N!}{V}\left(\frac{4\pi mE}{3N\hbar^2}\right)^{\frac{3}{2}} + \frac{3N}{2} \tag{19}$$

and the microcanonical partition function for fixed total energy and number of not interacting particles, can write in the form

$$\Omega_{E,N} = \exp\left\{-\ln\frac{N!}{V}\left(\frac{3N\hbar^2}{4\pi mE}\right)^{\frac{3}{2}} + \frac{3N}{2}\right\} \tag{20}$$

which reproduce well-know presentation microcanonical partition function for ideal Boltzmann gas:

$$\Omega_{E,N} = \frac{V^N}{N!}\left\{\frac{4\pi mEe^N}{3N\hbar^2}\right\}^{\frac{3N}{2}} \tag{21}$$

In the case homogeneous distribution particles $\nabla\varphi = 0$ with average distance between them $l = (\frac{V}{N})^{\frac{1}{3}}$ can introduce the average value of potential

$$\varphi_h = \frac{3Gm^2N}{2E}(\frac{N}{V})^{\frac{1}{3}} \tag{22}$$

and determine the ordinary entropy in the form

$$S_{E,N} = \ln\frac{N!}{Ve^{\varphi_h}}\left(\frac{4\pi mE}{3N\hbar^2}\right)^{\frac{3}{2}} + \frac{3N}{2} \tag{23}$$

The microcanonical partition function can obtain in the form:

$$\Omega_{E,N} = \frac{V^N}{N!}e^{-N\widetilde{\varphi}}\left\{\frac{4\pi mEe^N}{3N\hbar^2}\right\}^{\frac{3N}{2}} \tag{24}$$

Microcanonical partition function solve problem determination thermodynamically properties of the self-gravitational system in the case homogeneous distribution of particle. From entropy can determine the pressure in self gravitational system from homogeneous distribution of particle as $P_h = \frac{N}{\beta V e^{\varphi_h}}$ or equation of state in the form

$$P_h V_h = \frac{2E}{3e^{\varphi_h}} \tag{25}$$

But in general case the distribution of particle in self- gravitational system are inhomogeneous. Inhomogeneous distribution of particle motivate the long-range gravitational interaction. This system are unstable and all system divide as finite state into few cluster finite size. The next task are in developing possible method to determine partition function taking into account the inhomogeneous distribution with formation cluster of finite size. Let start with the situation when homogeneous distribution particle decomposing to n equal cluster with average volume V_c. Inside this volume exist nonhomogeneous distribution of particle with nonzero field variable, and outside this volume the field variable are zero because the particle are absence.

In this case the effective entropy can rewrite in the form:

$$S_{eff}\left(\varphi,\xi\right) = n\int_0^{V_c} dV\left\{\frac{1}{4r_m}(\nabla\varphi)^2 - \xi A e^{\varphi}\right\} - \xi A(V - nV_c) + N\ln\xi - \beta E \tag{26}$$

where $r_m = 2\pi G m^2\beta$ and $A = \left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}}$ as previously. Minimization of effective entropy on field variables lead to next saddle point solutions of equation inside the cluster:

$$\frac{1}{2r_m}\Delta\varphi + \xi A e^{\varphi} = 0 \tag{27}$$

and normalization condition yields:

$$n\int_0^{V_c} dV \xi A e^{\varphi} + \xi A(V - nV_c) = N \tag{28}$$

To multiply the first equation on $\nabla\varphi$ and used relation $\Delta\varphi = \nabla(\nabla\varphi)$ can obtain the first integral of this equation in the form:

$$\frac{1}{4r_m}(\nabla\varphi)^2 + \xi A e^{\varphi} = \Delta^2 \tag{29}$$

where Δ is unknown integral of "motion" which must determine from physical condition. If use the relation $\beta\frac{dA}{d\beta} = -\frac{3}{2}A$ the saddle point equation $\frac{\delta(S)}{\delta\beta} = 0$ can rewrite in the form

$$n\int_0^{V_c} dV\left\{\frac{5}{2}\xi A e^{\varphi} - \Delta^2\right\} + \frac{3}{2}\xi A(V - nV_c) = \beta E \tag{30}$$

If introduce the density function in the form $\rho(r) \equiv \xi A e^{\varphi}$ can rewrite the saddle point equations in the simple form normalization condition

$$n\int dV\rho(r) + \xi A(V - nV_c) = N \tag{31}$$

and equation for conservation energy

$$n\int dV\left\{\frac{5}{2}\rho(r) - \Delta^2\right\} + \frac{3}{2}\xi A(V - nV_c) = \beta E \tag{32}$$

The usual entropy can present in the simple form

$$S = \frac{1}{2}N - 2\beta E + N\ln\xi \tag{33}$$

For presentation the entropy in term know parameter we must determine unknown reverse temperature β and chemical activity ξ. The solution of obtained equation completely solve problem statistical description of self-gravitational

system, but in general case this solutions are unknown. To make an attempt to solve this problem in general case was take place in article [12]. Next will be present a certain solution of self-gravitational system in the case formation few cluster same size close pacing particle in ones. This condition correspond the final state of self-gravitational system. The field variable inside of cluster are constant and can be present as potential between two close pacing particle $\varphi = \varphi_0 = \frac{2\pi G m^2 \beta}{2R}$ for $r - r' = 2R$ where R is size of particle. For that is possible used the asymptotic value of field variable in center of cluster and determine first integral as $\Delta^2 \equiv \xi A e^{\varphi_0}$. The initial size of cluster can be determine from simple reason. In final case can assume that all particle assemble only to n clusters and take into account the finite size of particle which occupied volume $V_0 = \frac{4\pi}{3} R^3$ can estimate $nV_c \simeq NV_0$. The normalization condition in this case can present as

$$n\xi A e^{\varphi_0} + \xi A(V - nV_c) = N \tag{34}$$

and equation of conservation energy as

$$\frac{5}{2} n\xi A e^{\varphi_0} - nV_c \xi A e^{\varphi_0} + \frac{3}{2}\xi A(V - nV_c) = E \tag{35}$$

from which can obtain the chemical activity

$$\xi A = \frac{N}{V - nV_c(1 - e^{\varphi_0})} \tag{36}$$

and relation $\beta E = \frac{3}{2}N$. The usual entropy can rewrite in the other simple form

$$S = -N + N\ln\xi - \beta E = -N + N\ln\xi - \frac{3N}{2} \tag{37}$$

Substitution obtained relation to effective entropy yields:

$$S = -N + N\ln\frac{N}{A(V - nV_c(1 - e^{\varphi_0}))} - \frac{3N}{2} \tag{38}$$

This presentation fully solve problem the statistical description of self gravitational system with formation few cluster equal size. Using identity $N - NlnN \approx lnN!$ as result can obtain the ordinary entropy in the form

$$S^{inh}_{E,N} = \ln\frac{V^N}{N!}\left\{\frac{4\pi m E e^N}{3N\hbar^2(1 - \frac{NV_0}{V}(1 - e^{\varphi_0}))}\right\}^{\frac{3N}{2}} + \frac{3}{2}N \tag{39}$$

and the partition function microcanonical ensemble for self-gravitational system can present as:

$$Z_{E,N} = \frac{V^N}{N!}\left\{\frac{4\pi m E e^N}{3N\hbar^2(1 - \frac{NV_0}{V}(1 - e^{\varphi_0}))}\right\}^{\frac{3N}{2}} \tag{40}$$

If are not gravitational interaction between particles, than $e^{\varphi_0} = 1$ and the partition function reduces to partition function of ideal Boltzmann gas of hard sphere. The equation of state in the case inhomogeneous distribution of particle (existence few cluster finite size) can present in the form

$$P_{inh}V_{inh} = \frac{2E}{3(1 - \frac{NV_0}{V}(1 - e^{\varphi_0}))} \tag{41}$$

After this calculation can contend that the entropy of inhomogeneous distribution of particle (existence few cluster finite size) is bigger as entropy homogeneous distribution of particle

$$S_{inh} - S_h = \ln\frac{e^{\varphi_h}}{(1 - \frac{NV_0}{V}(1 - e^{\varphi_0}))} \tag{42}$$

if $e^{\varphi_h} > 1 - \frac{NV_0}{V}(1 - e^{\varphi_0})$ that take place for real self gravitational system. The homogeneous distribution of particle to meet the requirements of equilibrium state.This is the thermodynamically reason formation inhomogeneous

distribution of particle. The relation between ordinary entropy self gravitational system produce the next relation between equation of state

$$\frac{P_{inh}V_{inh}}{P_hV_h} = \frac{e^{\varphi_h}}{(1 - \frac{NV_0}{V}(1 - e^{\varphi_0}))} > 1 \tag{43}$$

If assume that the pressure in both states is equal, come to a determination that the volume of inhomogeneous distribution of particle of same self-gravitational system is bigger as volume of homogeneous distribution of particle. If the domains is unlimited the density of states diverges when the particles are spread to infinity.. Therefore, there is no equilibrium state in strict sense. Self-gravitational system have tendency to disperse. This is already the case for ordinary gas in infinite volume. The disperse rate is small in general and the system can be found in quasi equilibrium state for a relatively long time.

Indeed,present equilibrium statistical description tell only dilute structure in the self-gravitational system but not describe meta stable states and tell nothing about time scales a kinetic theory. The partition function have not any peculiarity for different value of gravitational field. The problem of description of the self-gravitational system of particles could be solved with current approach where entropy for finite system could be explicitly calculated. Spatial non-uniformity of particles as the equilibrium state might alter necessary activation barrier to proceed with transformation when the system is being moved into non-equilibrium state. Gravity factor could either promote or retard such transformation depending on the system and conditions concerned.

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